



**USING POWERFUL INSTRUCTIONAL
STRATEGIES AND TECHNIQUES TO
INCREASE MATHEMATICAL LEARNING
FOR STRUGGLING STUDENTS,
INCLUDING STUDENTS WITH
DISABILITIES**

**WEBINAR SERIES
SPONSORED BY**

**NEW JERSEY DEPARTMENT OF EDUCATION
OFFICE OF SPECIAL EDUCATION PROGRAMS
FUNDED BY IDEA PART B**



**FACILITATING FLUENCY AND
AUTOMATICITY THROUGH
PURPOSEFUL, PLANNED, AND
TARGETED PRACTICE
ACTIVITIES**

WEBINAR

Presented by
Paul J. Riccomini, Ph.D.

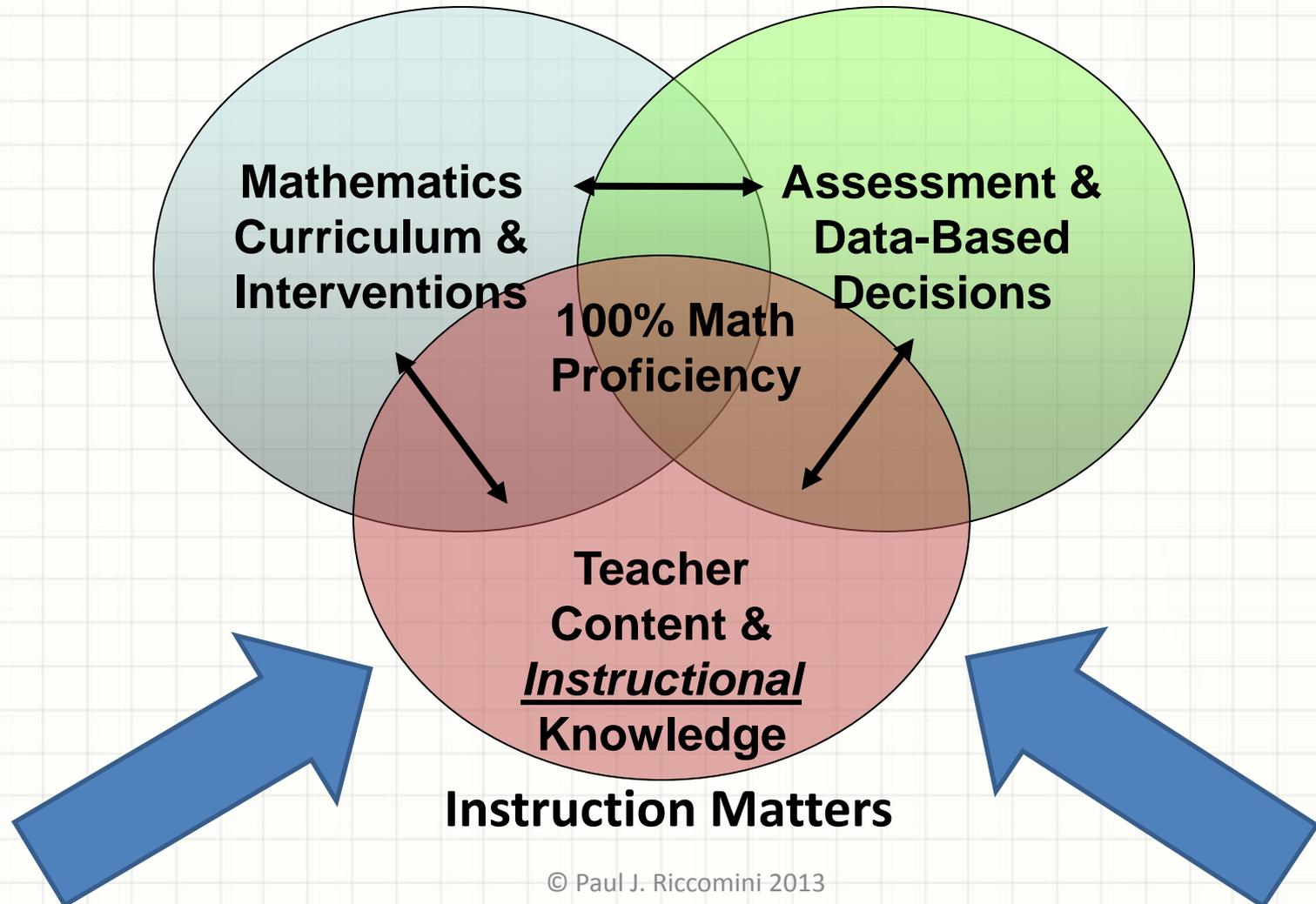
pjr146@psu.edu

 @pjr146

Topics for Today

- Overall purpose is to improve student learning outcomes in mathematics through the use of more effective instructional techniques
- NMAP, 2008 Final Report-OVERVIEW
 - Short-term working memory limits
- Fluency and Automaticity Strategies
 1. Single Digit Arithmetic Facts
 2. Computational & Procedural Fluency
- Conclusion

Components of Effective Mathematics Programs





Foundations for Success

National Mathematics Advisory Panel

Final Report, March 2008

Select Slides taken from the NMAP-Final Report
Presentation available at: <http://www.ed.gov/MathPanel>

Curricular Content

Streamline the Mathematics Curriculum in Grades PreK-8:

- Follow a *Coherent Progression*, with Emphasis on *Mastery of Key* Topics
- Focus on the Critical Foundations for Algebra
 - *Proficiency with Whole Numbers*
 - *Proficiency with Fractions*
 - Particular Aspects of *Geometry and Measurement*
- Avoid Any Approach that Continually Revisits Topics without Closure (pg 22)

Curricular Content

Streamline the Mathematics Curriculum in Grades PreK-8:

- Follow a Coherent Progression, with Emphasis on Mastery of Key Topics
****Enrichment & Reteaching Targets****

- Focus on the Critical Foundations for Algebra
 - Proficiency with Whole Numbers
 - Proficiency with Fractions
 - Particular Aspects of Geometry and Measurement

- Avoid Any Approach that Continually Revisits Topics without Closure (pg 22)

Learning Processes

- To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, factual knowledge and problem solving skills
- Limitations in the ability to keep many things in mind (working-memory) can hinder mathematics performance
 - Practice can offset this through automatic recall, which results in less information to keep in mind and frees attention for new aspects of material at hand
 - Learning is most effective when practice is combined with instruction on related concepts
 - Conceptual understanding promotes transfer of learning to new problems and better long-term retention

NMAP, 2008

Learning Processes

- To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, factual knowledge and problem solving skill.

- Limitations in the ability to keep many things in mind (working-memory) can hinder mathematics performance
 - Practice can offset this through automatic recall, which results in less information to keep in mind and frees attention for new aspects of material at hand
 - Learning is most effective when practice is combined with instruction on related concepts
 - Conceptual understanding promotes transfer of learning to new problems and better long-term retention

NMAP, 2008

Instructional Practices

Research on students who are low achievers, have difficulties in mathematics, or have learning disabilities related to mathematics tells us that the effective practice includes:

- ✓ Explicit methods of instruction available on a regular basis
- ✓ Clear problem solving models
- ✓ Carefully orchestrated examples/ sequences of examples.
- ✓ Concrete objects to understand abstract representations and notation.
- ✓ Participatory thinking aloud by students and teachers

For More Information

Please visit us online at:

<http://www.ed.gov/MathPanel>

- **Read it! Fact Sheet**
- **The report and fact sheet should be on the desk of every teacher responsible for teaching and planning math**

3-Minute Breakout Activity

- If you are in a group participating in this webinar, pause the webinar and take 3-5 minutes to discuss the Big Ideas covered to this point:
 - NMAP Final Report Overview
 - Working Memory Limitations and your current students
 - Do you see evidence of students who have greater limits in their working memory compared to other students.
 - Do you see large numbers of students (more than 35%) counting on their fingers for basic arithmetic?

Essential Question for Teachers

- Essential Question
 - What did I do *“instructionally different”* to support learning for the struggling students?
- Asked during instructional planning and after instructional delivery!!!

Strategies & Techniques

1

- **Fluency & Automaticity**

Fluency & Automaticity

- Learning Progression Stages
 - Understanding
 - Relationship
 - Fluency
 - Automaticity
- Fluency & Automaticity
 - Basic facts
 - Computation

Fluency & Automaticity

- Learning Progression Stages
 - **Understanding—Manipulatives & Pictorial Reps**
 - Relationship—
 - Fluency—
 - Automaticity—
- Fluency & Automaticity
 - Basic facts
 - Computation

Fluency & Automaticity

- Learning Progression Stages
 - Understanding—Manipulatives & Pictorial Reps
 - **Relationship—Making connections within & across**
 - Fluency
 - Automaticity—
 - Fluency & Automaticity
 - Basic facts
 - Computation

Fluency & Automaticity

- Learning Progression Stages
 - Understanding—Manipulatives & Pictorial Reps
 - Relationship—Making connections within & across
 - **Fluency—Strategy development for accuracy**
 - Automaticity—
- Fluency & Automaticity
 - Basic facts
 - Computation

Fluency & Automaticity

- Learning Progression Stages
 - Understanding—Manipulatives & Pictorial Reps
 - Relationship—Making connections within & across
 - Fluency—Strategy development for accuracy
 - **Automaticity— Practice to facilitate automaticity**
- Fluency & Automaticity
 - Basic facts
 - Computation

Why learn Facts?

1. Knowledge of simple facts is needed for proper use of calculators
2. Ability to estimate implies mastery of single digit facts
3. Students slow at facts are less likely to learn more complex math problem types
4. Students must know multiplication facts quickly to be able to master fractions
5. Algebra is not open to those who haven't mastered fractions

Crawford, 2002

Common Core State Standards

- Operations & Algebraic Thinking K.OA.A.5
 - Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
 - **Fluently** add and subtract within 5
- Operations & Algebraic Thinking 2.OA.2
 - 2. Fluently add and subtract within 20 using mental strategies
 - By end of Grade 2, **know from memory** all sums of two one-digit numbers

Common Core State Standards

- Operations & Algebraic Thinking 3.0A.7
 - **Fluently** multiply and divide within 100 using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40/5 = 8$ or properties of operations
 - By the end of Grade 3: know from memory all products of two one-digit numbers
- Number & Operation Base Ten 3.NBT
 - Use place understanding to round numbers to the nearest 10 or 100
 - Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction

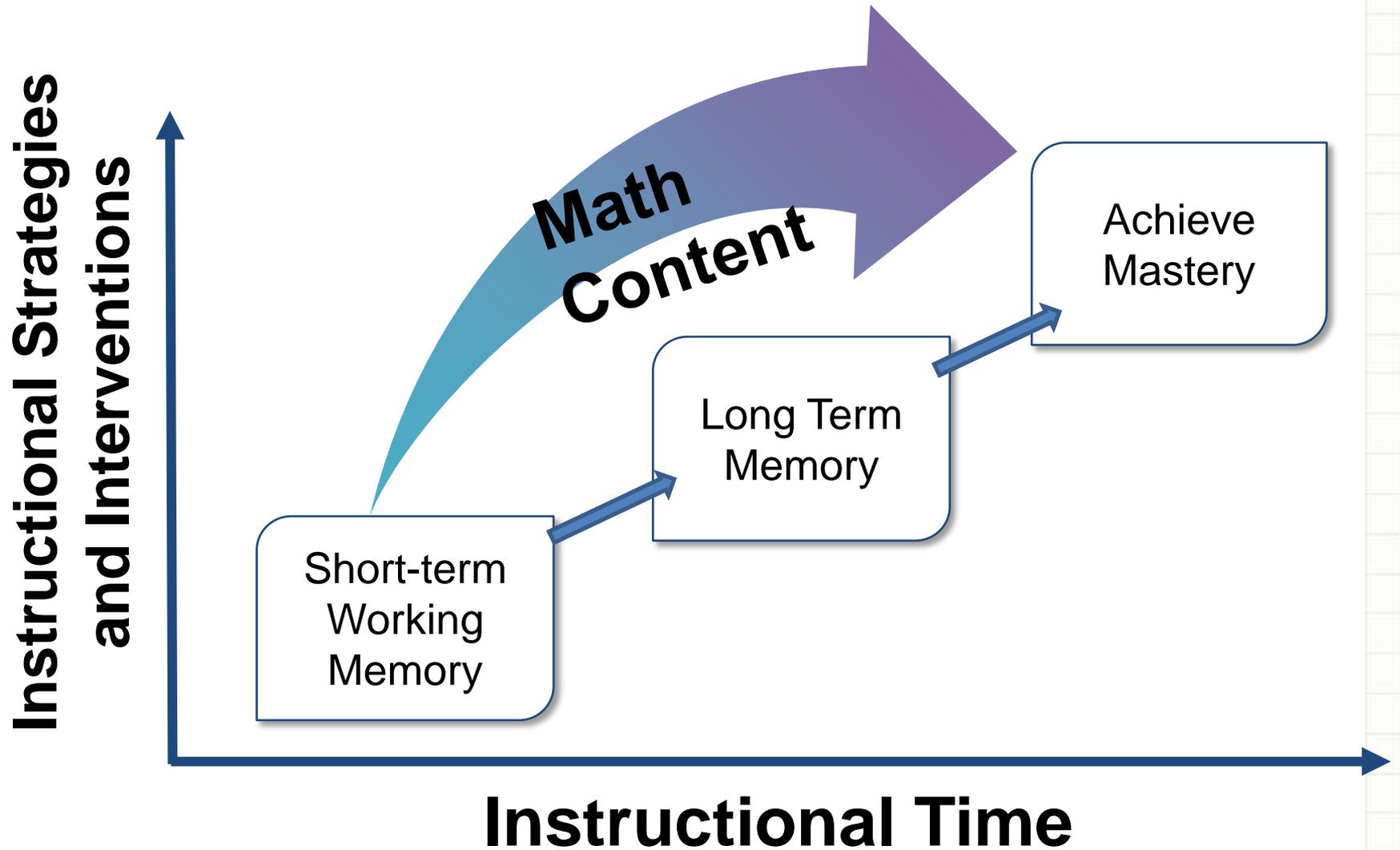
Common Core State Standards

- 5.NBT.5. Perform operations with multi-digit whole numbers and with decimals to hundredths
 - **Fluently** multiply multi-digit whole numbers using the standard algorithm
- 6.NS.2 Compute **fluently** with multi-digit numbers and find common factors
 - **Fluently** divide multi-digit numbers using the standard algorithm
- 6.NS.3. Compute **fluently** with multi-digit numbers and find common factors and multiples

Common Core State Standards

- **7.EE.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities
 - Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms ***fluently***. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Progression to Automaticity



Fluency/Automaticity

- Fluency and automaticity is needed for single digit facts as well as beyond the basic facts
- Clearly articulated in the Core Standards
- Mass practice or Drill-n-Kill is NOT an efficient or targeted technique for many struggling students
- Most practice efforts are **NOT FLUENCY** based...MORE ACCURACY BASED

What I See being used

- Fluency and Automaticity?
 - Drill and Kill
 - Games
 - Mad minutes
 - Flash cards
 - Parents- “Do it at Home”
 - Computers
 - Give calculator
 - others

???

**Are these
activities
PURPOSEFUL
PLANNED &
TARGETED?**

What I See being used

- Fluency and Automaticity?

- Drill and Kill- **NO**
- Games—**USUALLY NOT/COULD BE**
- Mad minutes – **NOT REALLY**
- Flash cards – **COULD BE**
- Parents- “Do it at Home”--**?????**
- Computers – **VERDICT STILL OUT**
- Give calculator – **DEFINITELY NOT**
- Others – **NOT SURE**

???

**Are these
activities
PURPOSEFUL
PLANNED &
TARGETED?**

Grade Level Planning Questions

- **Fluency/Automaticity**

- How many students in your current classroom AND school are not automatic with basic facts?

Grade 1-Addition

Grade 3-4: Multiplication & Division

Grade 2-Subtraction

Grade 4-5+: Multiplication is Priority

- How are you currently helping student build fluency and automaticity with basic math facts...ALL 4 Operations

- 4th Grade+ focus on Multiplication?

- How are these current efforts to facilitate fluency/automaticity working?

Steps to Fluency & Automaticity

Requires

1. Specific criterion for introducing new facts
2. Intensive practice on newly introduced facts (more than 1x)
3. Systematic practice on previously introduced facts
4. Adequate allotted time (5-10 min/day)
5. Record keeping
6. Motivational procedures

Steps to Fluency & Automaticity

Requires

3-4 Facts



1. Specific criterion for introducing new facts
2. Intensive practice on newly introduced facts (more than 1x)
3. Systematic practice on previously introduced facts
4. Adequate allotted time (5-10 min/day)
5. Record keeping
6. Motivational procedures

Steps to Fluency & Automaticity

Requires

1. Specific criterion for introducing new facts
2. Intensive practice on newly introduced facts (more than 1x)
3. Systematic practice on previously introduced facts
4. Adequate allotted time (5-10 min/day)
5. Record keeping
6. Motivational procedures

3-4 Facts



Each fact
on 4 cards



Steps to Fluency & Automaticity

Requires

1. Specific criterion for introducing new facts
2. Intensive practice on newly introduced facts (more than 1x)
3. Systematic practice on previously introduced facts
4. Adequate allotted time (5-10 min/day)
5. Record keeping
6. Motivational procedures

3-4 Facts



Each fact
on 4 cards



Continuous
Cycle



Steps to Fluency & Automaticity

Requires

1. Specific criterion for introducing new facts
2. Intensive practice on newly introduced facts (more than 1x)
3. Systematic practice on previously introduced facts
4. Adequate allotted time (5-10 min/day)
5. Record keeping
6. Motivational procedures

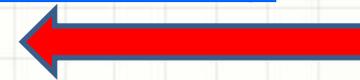
3-4 Facts



Each fact
on 4 cards



Continuous
Cycle



Step #1 & #2
4 New facts on
4 flashcards each

New Fact #1	New Fact #1
New Fact #1	New Fact #1

New Fact #2	New Fact #2
New Fact #2	New Fact #2

New Fact #3	New Fact #3
New Fact #3	New Fact #3

New Fact #4	New Fact #4
New Fact #4	New Fact #4

Step 3
Previously
learned fact

PL Fact A

PL Fact B

PL Fact C

PL Fact D

Cycle Continues: After Students are automatic on New Facts 1-4

Step #1 & #2
4 New facts on
4 flashcards each

Step 3
Previously
learned fact

Step #1 & #2
4 New facts on
4 flashcards each

New Fact #1	New Fact #1
New Fact #1	New Fact #1

New Fact #2	New Fact #2
New Fact #2	New Fact #2

New Fact #3	New Fact #3
New Fact #3	New Fact #3

New Fact #4	New Fact #4
New Fact #4	New Fact #4

Step 3
Previously
learned fact

PL Fact A

PL Fact B

PL Fact C

PL Fact D

Cycle Continues: After Students are automatic on New Facts 1-4

Step #1 & #2
4 New facts on
4 flashcards each

New Fact #5	New Fact #5
New Fact #5	New Fact #5

New Fact #6	New Fact #6
New Fact #6	New Fact #6

New Fact #7	New Fact #7
New Fact #7	New Fact #7

New Fact #8	New Fact #8
New Fact #8	New Fact #8

Step 3
Previously
learned fact

New Fact #1	New Fact #1
-------------	-------------

New Fact #2	New Fact #2
-------------	-------------

New Fact #3	New Fact #3
-------------	-------------

New Fact #4	New Fact #4
-------------	-------------

Cycle Continues: After Students are automatic on New Facts 5-8

Step #1 & #2
4 New facts on
4 flashcards each

New Fact #9	New Fact #9
New Fact #9	New Fact #9

New Fact #10	New Fact #10
New Fact #10	New Fact #10

New Fact #11	New Fact #11
New Fact #11	New Fact #11

New Fact #12	New Fact #12
New Fact #12	New Fact #12

Step 3
Previously
learned fact

New Fact #5	New Fact #5
-------------	-------------

New Fact #6	New Fact #6
-------------	-------------

New Fact #7	New Fact #7
-------------	-------------

New Fact #8	New Fact #8
-------------	-------------

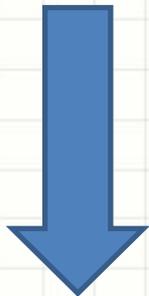
New Fact #1

New Fact #2

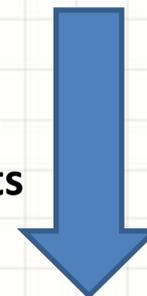
New Fact #3

New Fact #4

Cycle Continues: as students become automatic with newly introduced facts



**Continue Adding New Facts and
Reducing Previously Learned facts**



Steps to Fluency & Automaticity

Requires

1. Specific criterion for introducing new facts
2. Intensive practice on newly introduced facts (more than 1x)
3. Systematic practice on previously introduced facts
4. Adequate allotted time (5-10 min/day)
5. Record keeping
6. Motivational procedures

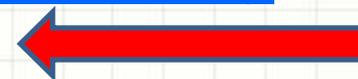
3-4 Facts



Each fact
on 4 cards



Continuous
Cycle



Regular
Schedule



Steps to Fluency & Automaticity

Requires

1. Specific criterion for introducing new facts  **3-4 Facts**
2. Intensive practice on newly introduced facts (more than 1x)  **Each fact on 4 cards**
3. Systematic practice on previously introduced facts  **Continuous Cycle**
4. Adequate allotted time (5-10 min/day)
5. Record keeping  **Regular Schedule**
6. Motivational procedures

Practice Procedures Math Facts

- Peer to Peer or individual or small group
- Students must say the fact (4 x 5 is 20)
 - “Four times five is twenty”
- Error Correction Procedure:
 - The only correct response is the correct answer to the fact
 - All other responses should be corrected
 - For example....Saying incorrect fact, hesitation, using a strategy
 - 1. Stop student and say correct answer (I say)
 - 2. Say correct answer with student (We say)
 - 3. Have student say correct answer (You say)
 - 4. Partner says correct answer (I say)
 - Fact is placed three cards back to make sure student has opportunity to re-practice the fact while the correction is still in short term memory

3-Minute Breakout Activity

- If you are in a group listening to the pre-recorded webinar, pause the webinar and take 3-5 minutes to discuss the Big Ideas covered to this point:
 - Number of students who are NOT fluent
 - Discuss current activities that teachers are doing for fluency
 - Discuss the 6 Steps and how to implement with flash cards or in other activities

Automaticity Practice Activities

Beyond basic arithmetic:

- Fluency/automaticity in other mathematic computations and procedures becomes important for the transition to Algebra
 - Computational Fluency
 - Procedural Fluency

Fluency & Automaticity Activities

The Math Dash Activity

1. Explain fluency and the purpose of fluency to your students
2. Target a Skill that students have already learned & are accurate (Rule of thumb 85%-90%)
3. Create a PowerPoint with 8-10 problems in that specific skill area
4. Display problems one at a time at a predetermined rate appropriate for age and targeted skill (e.g., 3 seconds...). Vary rate as necessary
5. Only ONE problem at a time is displayed and students are required to just right the answer
6. Display answers—check students for accuracy
7. Repeat 2-3 times for several days

Round to Nearest
Whole Number

96.39 = 96
37.99 = 38
50.91 = 51
31.29 = 31
1.83 = 2
1.48 = 1

Solve:

$(-7) - 2 = -9$
 $(-8) - 3 = -11$
 $-6 + 8 = 2$
 $(-5) + 1 = -4$
 $+9 - (-4) = 13$
 $(+9) - 7 = 2$

Fluency & Automaticity Activities

- The Math Dash Activity

Opener Activity for Fluency and Automaticity

Round to Nearest Whole Number 50.91 =	Solve: $-6 + -8 =$
--	-----------------------

Opener Activity for Fluency and Automaticity

Round to Nearest Whole Number 31.29 =	Solve: $(-5)(+1) =$
--	------------------------

Opener Activity for Fluency and Automaticity

Round to Nearest Whole Number 1.83 =	Solve: $+9 - (-4) =$
---	-------------------------

Opener Activity for Fluency and Automaticity

Round to Nearest Whole Number 1.48 =	Solve: $(+9)(-7) =$
---	------------------------

Opener Activity for Fluency and Automaticity

Round to Nearest Whole Number 96.39 = <u>96</u> 37.99 = <u>38</u> 50.91 = <u>51</u> 31.29 = <u>31</u> 1.83 = <u>2</u> 1.48 = <u>1</u>	Solve: $(+7)(-2) = \underline{-14}$ $(-8)(-3) = \underline{+24}$ $-6 + -8 = \underline{-14}$ $(-5)(+1) = \underline{-5}$ $+9 - (-4) = \underline{+13}$ $(+9)(-7) = \underline{-63}$
---	---

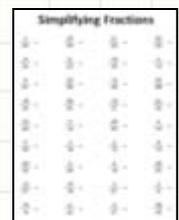
Opener Activity for Fluency and Automaticity

Round to Nearest Whole Number 96.39 = <u>96</u> 37.99 = <u>38</u> 50.91 = <u>51</u> 31.29 = <u>31</u> 1.83 = <u>2</u> 1.48 = <u>1</u>	Solve: $(+7)(-2) = \underline{-14}$ $(-8)(-3) = \underline{+24}$ $-6 + -8 = \underline{-14}$ $(-5)(+1) = \underline{-5}$ $+9 - (-4) = \underline{+13}$ $(+9)(-7) = \underline{-63}$
---	---

Fluency & Automaticity Activities

- **The Method of Repeated Calculation**

1. Explain fluency and the purpose of fluency to your students—especially that students are not expected to answer ALL of the problems
 - Goal is to solve more each time—even if that is one more
2. Select target skill and create a worksheet with around 30 problems (depends on targeted skill). Remember—Already learned skill
3. Students are given 1 minute to solve as many problems as they can. Students can skip problems they don't know and go to the next.
 - DO NOT ALLOW SKIPPING AROUND THE PAGE
4. Repeat this activity 2-3 times (no more than 3) for several days—use the same problems
5. Students can track their progress



Fluency & Automaticity Activities

- The Method of Repeated Calculation

$\frac{4}{24} =$	$\frac{25}{35} =$	$\frac{21}{30} =$	$\frac{20}{24} =$
$\frac{9}{24} =$	$\frac{5}{20} =$	$\frac{10}{35} =$	$\frac{4}{14} =$
$\frac{9}{21} =$	$\frac{32}{36} =$	$\frac{15}{35} =$	$\frac{20}{28} =$
$\frac{15}{24} =$	$\frac{55}{60} =$	$\frac{15}{27} =$	$\frac{21}{30} =$
$\frac{20}{28} =$	$\frac{3}{12} =$	$\frac{45}{50} =$	$\frac{4}{10} =$
$\frac{5}{45} =$	$\frac{3}{18} =$	$\frac{2}{10} =$	$\frac{8}{14} =$
$\frac{20}{45} =$	$\frac{4}{28} =$	$\frac{4}{18} =$	$\frac{45}{50} =$
$\frac{12}{27} =$	$\frac{15}{18} =$	$\frac{3}{21} =$	$\frac{2}{6} =$
$\frac{10}{14} =$	$\frac{4}{32} =$	$\frac{3}{27} =$	$\frac{35}{45} =$

$\frac{4}{24} =$	$\frac{25}{35} =$	$\frac{21}{30} =$	$\frac{20}{24} =$
$\frac{9}{24} =$	$\frac{5}{20} =$	$\frac{10}{35} =$	$\frac{4}{14} =$
$\frac{9}{21} =$	$\frac{32}{36} =$	$\frac{15}{35} =$	$\frac{20}{28} =$
$\frac{15}{24} =$	$\frac{55}{60} =$	$\frac{15}{27} =$	$\frac{21}{30} =$
$\frac{20}{28} =$	$\frac{3}{12} =$	$\frac{45}{50} =$	$\frac{4}{10} =$
$\frac{5}{45} =$	$\frac{3}{18} =$	$\frac{2}{10} =$	$\frac{8}{14} =$
$\frac{20}{45} =$	$\frac{4}{28} =$	$\frac{4}{18} =$	$\frac{45}{50} =$
$\frac{12}{27} =$	$\frac{15}{18} =$	$\frac{3}{21} =$	$\frac{2}{6} =$
$\frac{10}{14} =$	$\frac{4}{32} =$	$\frac{3}{27} =$	$\frac{35}{45} =$

Grade Level Planning Activity

- **Computational/Procedural Fluency**

- Select an upcoming Chapter/Unit; one that you are teaching within the next 2-3 weeks.
- Review the content (Big ideas) for that unit and identify important fluency skills that are important (or helpful) to learning the main goals of that unit
- Discuss how you might facilitate students' fluency through the activities that you just learned.

Facilitating Fluency & Automaticity

- Automaticity of facts is vital, but instruction for conceptual understanding must occur first
- Automaticity activities must be cumulative
 - Newly introduced facts receive intensive practice, while previously introduced facts receive less intensive, but still **SYSTEMATICALLY PLANNED**.
- Fluency/Automaticity building activities should **NOT** use up all of the allocated math time (less than 10 minutes).
- Fact **automaticity instruction** is often **overlooked** by most math programs or ill-conceived.
- Automaticity practice must be purposeful and systematic as well as carefully controlled by the teacher



Conclusion

- The learning needs of struggling students and students with disabilities in mathematics is extremely challenging for teachers
- Effective teachers always consider the Task and the requirements placed on students working memory
- Fluency and automaticity activities that are purposeful, planned, and targeted are essential
 - 6 Steps to Automaticity
 - Math Dash Activity
 - Method of Repeated Calculation

QUESTIONS?

Contact Information:

Paul J. Riccomini

pjr146@psu.edu

 @pjr146

