

## Learning PATHS and Teaching STRATEGIES in Early Mathematics

The research base for sketching a picture of children's mathematical development varies considerably from one area of mathematics to another. Outlining a learning path, moreover, does not mean we can predict with confidence where a child of a given age will be in that sequence. Developmental variation is the norm, not the exception. However, children do tend to follow similar *sequences, or learning paths*, as they develop. This chart illustrates in each area some things that *many* children know and do—

early and late in the 3–6 age range. These are, then, simply two points along the learning path that may have many steps in between. For each content area, the Sample Teaching Strategies column shows a few of the many teacher actions that promote learning when used within a classroom context that reflects the recommendations set forth in this NAEYC/NCTM position statement. In general, they are helpful strategies, with minor adaptations, across the age range.

Content Area	Examples of Typical Knowledge and Skills		Sample Teaching Strategies
	From Age 3	Age 6	
Number and operations	Counts a collection of one to four items and begins to understand that the last counting word tells <i>how many</i> .	Counts and produces (counts out) collections up to 100 using groups of 10.	Models counting of small collections and guides children's counting in everyday situations, emphasizing that we use one counting word for each object:   "One...two...three..." Models counting by 10s while making groups of 10s (e.g., 10, 20, 30... or 14, 24, 34...).
	Quickly "sees" and labels collections of one to three with a number.	Quickly "sees" and labels with the correct number "patterned" collections (e.g., dominoes) and unpatterned collections of up to about six items.	Gives children a brief glimpse (a couple of seconds) of a small collection of items and asks how many there are.

Content Area	Examples of Typical Knowledge and Skills		Sample Teaching Strategies
	From Age 3	Age 6	
Number and operations	Adds and subtracts non-verbally when numbers are very low. For example, when one ball and then another are put into the box, expects the box to contain two balls.	Adds or subtracts using counting-based strategies such as counting on (e.g., adding 3 to 5, says "Five . . . , six, seven, eight"), when numbers and totals do not go beyond 10.	Tells real-life stories involving numbers and a problem. Asks <i>how many</i> questions (e.g., "How many are left?" "How many are there now?" "How many did they start with?" "How many were added?").  Shows children the use of objects, fingers, counting on, guessing, and checking to solve problems.
Geometry and spatial sense	Begins to match and name 2-D and 3-D shapes, first only with same size and orientation, then shapes that differ in size and orientation (e.g., a large triangle sitting on its point versus a small one sitting on its side).	Recognizes and names a variety of 2-D and 3-D shapes (e.g., quadrilaterals, trapezoids, rhombi, hexagons, spheres, cubes) in any orientation.  Describes basic features of shapes (e.g., number of sides or angles).	Introduces and labels a wide variety of shapes (e.g., skinny triangles, fat rectangles, prisms) that are in a variety of positions (e.g., a square or a triangle standing on a corner, a cylinder "standing up" or horizontal).  Involves children in constructing shapes and talking about their features.
	Uses shapes, separately, to create a picture.  	Makes a picture by combining shapes.  	Encourages children to make pictures or models of familiar objects using shape blocks, paper shapes, or other materials.  Encourages children to make and talk about models with blocks and toys.  Challenges children to mark a path from a table to the wastebasket with masking tape, then draw a map of the path, adding pictures of objects appearing along the path, such as a table or easel.
	Describes object locations with spatial words such as <i>under</i> and <i>behind</i> and builds simple but meaningful "maps" with toys such as houses, cars, and trees.	Buils, draws, or follows simple maps of familiar places, such as the classroom or playground.	

Content Area	Examples of Typical Knowledge and Skills		Sample Teaching Strategies
	From Age 3	→ Age 6	
Measurement	<p>Recognizes and labels measurable attributes of objects (e.g., "I need a long string," "Is this heavy?").</p> <p>Begins to compare and sort according to these attributes (e.g., <i>more/less, heavy/light</i>; "This block is too short to be the bridge").</p>	<p>Tries out various processes and units for measurement and begins to notice different results of one method or another (e.g., what happens when we <i>don't</i> use a standard unit).</p> <p>Makes use of nonstandard measuring tools or uses conventional tools such as a cup or ruler in non-standard ways (e.g., "It's three rulers long").</p>	<p>Uses comparing words to model and discuss measuring (e.g., "This book feels heavier than that block," "I wonder if this block tower is taller than the desk?").</p> <p>Uses and creates situations that draw children's attention to the problem of measuring something with two different units (e.g., making garden rows "four shoes" apart, first using a teacher's shoe and then a child's shoe).</p>
Pattern/ algebraic thinking	<p>Notices and copies simple repeating patterns, such as a wall of blocks with long, short, long, short, long, short, long....</p>	<p>Notices and discusses patterns in arithmetic (e.g., adding one to any number results in the next "counting number").</p>	<p>Encourages, models, and discusses patterns (e.g., "What's missing?" "Why do you think that is a pattern?" "I need a blue next"). Engages children in finding color and shape patterns in the environment, number patterns on calendars and charts (e.g., with the numerals 1-100), patterns in arithmetic (e.g., recognizing that when zero is added to a number, the sum is always that number).</p>
Displaying and analyzing data	<p>Sorts objects and counts and compares the groups formed.</p> <p>Helps to make simple graphs (e.g., a pictograph formed as each child places her own photo in the row indicating her preferred treat—pretzels or crackers).</p>	<p>Organizes and displays data through simple numerical representations such as bar graphs and counts the number in each group.</p>	<p>Invites children to sort and organize collected materials by color, size, shape, etc. Asks them to compare groups to find which group has the most.</p> <p>Uses "not" language to help children analyze their data (e.g., "All of these things are red, and these things are NOT red").</p> <p>Works with children to make simple numerical summaries such as tables and bar graphs, comparing parts of the data.</p>

